Predicting maize response to fertilizer application using growth curves in western Kenya

[1]Samuel K. Chomba, [1]John R. Okalebo, [1*]Moses Imo, [2]Patrick K. Mutuo and [3]Walsh G. Markus

[1]University of Eldoret, Kenya
[2]Millennium Village Project (ICRAF), Kisumu, Kenya
[3]The Earth Institute, Columbia University, USA

Abstract

To increase food security among smallholder farmers in western Kenya, which is one of the hunger hotspot in the country, evaluation of soil factors that determine yield of the major staple crops such as maize may enable development of site specific management regimes. In this study, we used Richards-Chapman plant growth function to examine the effect of N, P and K fertilization on maize growth and yield in nine randomly selected farms at Suari in Siaya district, western Kenya. Correlation analysis showed that maximum attainable growth rate is the most important predictor of maize grain yield (r^2 = 0.70; p= 0.035). The results also showed that combined N and P fertilizer application gave the highest maximum attainable maize stalk volume and growth rate. Applying N and P as single fertilizers had relatively similar growth responses, but lower than the N+P fertilizer application suggesting positive N+P interactions. These results indicate that maximum attainable stalk volume and growth rates are important indicators for measuring maize response to nutrient availability. The advantages of the multilevel nonlinear mixed effects model include its flexibility to model multiple sources of heterogeneity and complex patterns of correlation, and its higher power to make treatment comparisons.

Keywords: Soil productivity indexing, nonlinear growth models, fertilizer response curves.

INTRODUCTION

Although maize (*Zea mays*) is the most important staple food crop in western Kenya contributing about 40% - 50% of calories to the diet, its production in the area is only 0.8 – 1.4 t ha^{-1}, while the actual potential is four-fold mainly due to declining soil fertility (Smaling, 1993) and Striga (*Striga hermonthica*) weeds (Kiwia *et al.,* 2010). Other constraints include unreliable rainfall and shortage of land due to increasing land sub divisions among the growing families. Efforts by most farmers to restore soil fertility are inadequate since a few can afford to use blanket fertilizer application rates recommended for the area. At the same time, the use of manure is also restricted as farmers can only produce limited quantities that are inadequate to meet crop requirements.

Promoting the concept of "precision agriculture" in which land management is based on the understanding of the relationship between crop yield, land characteristics and soil parameters at specific sites (Machando *et al.,* 2002) holds key to increasing food production in the region. Such relationships can be evaluated using growth analysis techniques by assessing the independent and interactive effects of various factors affecting maize yields. This enables management of these factors in an integrated manner (Smith, 2001). Although site quality is governed by many independent and interacting factors e.g. moisture and nutrients, it is often difficult to assign cause and effects relationships even though establishment of this link is critical for site specific management. In growth analysis, the flexibility of the growth model in representing the growth pattern of plants is important such as allometric relations and actual metabolic rates and the biological meaning of the parameters estimated (Goudriaan and van Laar, 1994; Berzsenyi and Dang Q. Lap, 2004). From previous work some growth functions proved not sufficiently flexible to describe the variety of shapes represented in the plant growth data in the fields. The common disadvantage formula of Mitscherlich, Logistic, Gompertz is that the

reflection point is fixed, 0, A/2, A/e respectively. Beta function (Austin *et al.,* 1994) confounds the location of the maximum and the skewness (Oksanen, 1997). Richards (1959); Chapman (1961) both argued that Von Bertalanffy's (1939) allometric constant of 2/3 was too restrictive to apply to many forms of life. The Richards-Chapman sigmoid growth model provides a continuous transition from exponential phase, a linear phase; and a saturation phase for ripening and an asymmetrical growth as Weibull, Logistic and Gompertiz functions (Goudriaan and van Laar, 1994). In theoretical models, the Richards-Chapman function is valued for its accuracy in flexibility; that is inflection point of growth rate and (parameter *c*) can either be within the range of 0 to 1 or greater than 1). The function has been employed more than any other functions in studies of plant and stand growth (Zeide, 1993).

Fortunately, studies have indicated that soil fertility status correlates well with maize stalk height growth at which maximum grain yield is attained. This can be evaluated using the following Richards-Chapman growth function (Yin *et al*., 2003):

$$Y(t) = a\left(1 - \exp(-b * t)\right)^{c}$$

Where *Y(t)* is growth attained at any given time, *a* is the maximum attainable growth, *b* is the rate of growth of the crop, *c* is the shape of the curve, and *t* is the time (age). This curve is characteristically nonlinear in which crop growth and yield is evaluated as a function of age (Goudriaan and van Laar, 1994). The function has two opposite groups of growth factors i.e. an expansion phase representing the tendency to grow and multiply, and the decline phase representing a proxy of growth constraints such as ageing and growth limiting factor such as soil fertility. The first derivative of this function gives the inflection point at which time the highest growth rate is achieved (i.e. point of greatest slope on the curve). This analytical approach has been used widely in forestry research to study forest growth and yield (see for example Gregoire *et al.,* 1995; Fang and Bailey, 2001; Hall and Bailey, 2001).

The purpose of this study was to use this growth function to evaluate the effects of fertilizer application on maize growth and to predict maize grain yield. The objective was to determine the effects of the major plant nutrients (N, P and K) on maize growth characteristics, namely maximum growth attained, rate of growth, and shape of the growth curve. The intention was to identify the plant parameters that are the best indicators of soil productivity for efficient production of maize in western Kenya.

MATERIALS AND METHODS

Study sites

The experiment was carried out at Sauri area in western Kenya at an altitude of between 1400 – 1500m a.s.l. within the Lake Victoria basin (Joetzold and Schimdth, 1983). The general topography is undulating with ephemeral streams, rivers and wetlands, meandering through rounded hills. The area is classified as sub-humid tropics with an average temperature of 24ºC ranging from 18 to 27ºC, and an average annual rainfall of 1830 mm. Rainfall is bimodal, divided into the long rainy season from March to June (1120 mm), and the short rainy season from September to December (710 mm). The short rains are extremely variable but predictable. The main soils are classified as Oxisols/Nitisols (Kandiudalfic Eutrodox), and are mainly clayey, reddish, deep, and well drained. Derived from volcanic materials, the soils were once quite fertile but are now depleted of nitrogen (N) and phosphorus (P), two of the most limiting nutrients to plant growth in the region. Soil pH borders around 5.5, though soil acidity is not a major problem for plant growth. Soil carbon levels (1.3% C) are less than half those of the 'native' soils. There were some patches of wetland soils along the rivers and streams. Sauri sub location covers 7.6 km^2 and has ten villages with 1075 farms. The major land use is subsistence farming and most grown crops are maize and beans.

Experimental treatments and management

The sites were selected using a satellite image (Quick Bird Inc.) taken covering the whole area from which ten farms were selected using random stratification, one in every village. In each of the selected farms, two fertilizer test strips (one on the upslope and the second on low lying slope) were established for monitoring maize growth. In each block, eight 3m x 3m treatment plots were established for application of the various treatments.

The experiment had three-factors (N, P and K) and their various combinations as follows: N, P, K, NP, NK, PK, NPK plus a control without any fertilizer application in a randomized complete block design. Nitrogen was applied as urea (46% N); P as triple superphosphate (43-52 % P_2O_5 or 19-23 % P), and K as muriate of potash (50 % K). Since determination of recommended fertilizer rates was not an objective, high rates of N, P and K were used to assess the magnitude of nutrient responses rather than the economic rates of nutrient additions. All P and K additions were broadcast and incorporated at planting at the rate of 500 kg ha^{-1} of TSP and 200 kg ha^{-1} of KCl, which was equivalent to 100 kg P and K ha^{-1}, respectively. Nitrogen was applied as urea at the rate of 216 kg ha^{-1} (equivalent to 100 kg N ha^{-1}) in split application. One-third (73 kg ha^{-1}) was broadcast and incorporated at planting and the other two-thirds (143 kg ha^{-1}) was top dressed by banding and incorporation along the maize rows four weeks after emergence. Hybrid

maize seeds (H513) were planted at the spacing of 75 cm x 30 cm, at the rate of two per hole and later thinned to one per hole after germination giving a density of 4.4 plants m^{-2}. Weeding was done twice during the growing season; at four weeks after emergence and just before top dressing with urea, and at thirteen weeks just before tassling, which is the onset of the reproductive phase in maize.

Sampling

Hybrid maize seeds were planted. Five maize plants were randomly sampled from each treatment tagged and numbered for measurements over time, which represented 30% of the net harvested plants. The parameters measured were the total height and root collar diameter after every 3 weeks until tassling at week thirteen. The focus of the study was to compare responses of the maize stalk volume growth and height growth as indicators of site productivity, and for predicting the final yield (Mallarino *et al.,* 1999; Katsvairo *et al.,* 2003; Kyle *et al.,* 2005).

Maize harvesting was done by discarding the two outermost rows per treatment plot and two plants per row at the two ends of each treatment plot. Thus, two inner rows per treatment plot were harvested giving an effective area of 2.7 m^2. Cobs were picked and fresh weight of the cobs taken and arranged into different classes (big, medium and small), and a sub-sample of eight cobs were picked proportionately from each class. Fresh weight of each sub sample was taken and the cobs placed in paper bags and labeled. The cobs were oven dried at 60^0C for 72 hours to attain constant dry mass, and the grains hand shelled from the cobs and weighed to determine the grain dry matter factor. The maize stovers were cut at ground level, weighed for fresh weight. A sample of eight stalks was chopped into small pieces and mixed thoroughly for determination of dry mass after oven drying at 60^0C for 72 hrs. Yield calculations were done using the following expressions: (i) Dry matter factor [DMF] = (sample dry weight/sample fresh weight)/100; and (ii) Gain yield (GY, Kg/ha) = **{**(Total fresh weight * Dry matter factor)/Effective Area**}** * 10000. Root collar diameter (D) was measured at 3 cm above the prop root using Vanier calipers and subsequently used to calculate basal area (BA) and stalk volume (SV), such that BA = $(D/2)^2\pi$; and SV = BA*(Height).

Data analysis

All maize growth data was analyzed using the maximum likelihood nonlinear mixed-effect model (Peek *et al.,* 2002; Zhao *et al*, 2005) that accounted for both fixed (fertilizer treatments) and random (experimental unit) effects on maize growth functions over time. Thus, the above Richards-Chapman model was fitted using the nonlinear mixed effects model (NLME) library in S-Plus® software as described by Pinheiro and Bates (2000) using the following General nonlinear Model (NLME):

$$Ty_{jl}=Gm_{jl}\left\{1-\exp\left(-Rg_{jl}*Age\right)^{\hat{Cg}_{jl}}\right\}+\varepsilon_{jl}$$

where

$$Gm_{jl} = \beta_0 + \beta_{01}T_1 + \beta_{02}T_2 + \beta_{03}T_3 + \ldots \beta_{08}T_8 + b_j^{(0)}$$

$$Rg_{jl} = \beta_1 + \beta_{11}T_1 + \beta_{12}T_2 + \beta_{13}T_3 + \ldots \beta_{18}T_8 + bj^{(1)}$$

$$Cg_{jl} = \beta_2 + \beta_{21}T_1 + \beta_{22}T_2 + \beta_{23}T_3 + \ldots \beta_{28}T_8 + b_j^{(2)}$$

Where the *T* is the respective measured growth parameter (i.e. maximum growth attained [*Ty*], growth rate [*Rg*] or shape of the growth curve [*Cg*]); the β's are the fixed effects accounting for the treatment differences, while the plot-level (b_i) is the random effect accounting for the heterogeneity and implicitly account for within-plot correlation, however within-plot dependence among observations required some variance-covariance structure to describe the dependence. Also, *i* denotes the plot level effect = 10, and *l* is the time of measurement = 4, respectively.

RESULTS

Growth functions

The results of the study showed that the Richards-Chapman growth function represented the observed maize growth trend well as exemplified by the good agreement between the observed and the predicted plant growth and yield responses (Figure 1, r^2 = 0.956). The full fixed effect model included all the three parameters (maximum growth yield [*Ty*], rate of growth [*Rg*] and shape of growth curve [*Cg*]) as fixed effects, with random effects (plot-level random effects).

Modeling between plots variability

The model which included all the treatment effects for the three parameters (Ty,- maximum growth yield, Rg -rate of growth and Cg- shape of growth curve) as fixed effects (β's) was considered the full mixed model. The model had two levels of random effects: Block level and treatment plot-level random effects. The following assumptions were made (1) diagonal structures of variance-covariance matrices were assumed, (2) Random effects are normally distributed and independent for different groups and the within-group errors are independent and identically normally distributed and independent of the random effect. Comparison statistics between the full mixed model and a model that had random effects at block level only; likelihood ratio tests (LTR) of 2.334 with p=0.177, 3.456 with p=0.211 and

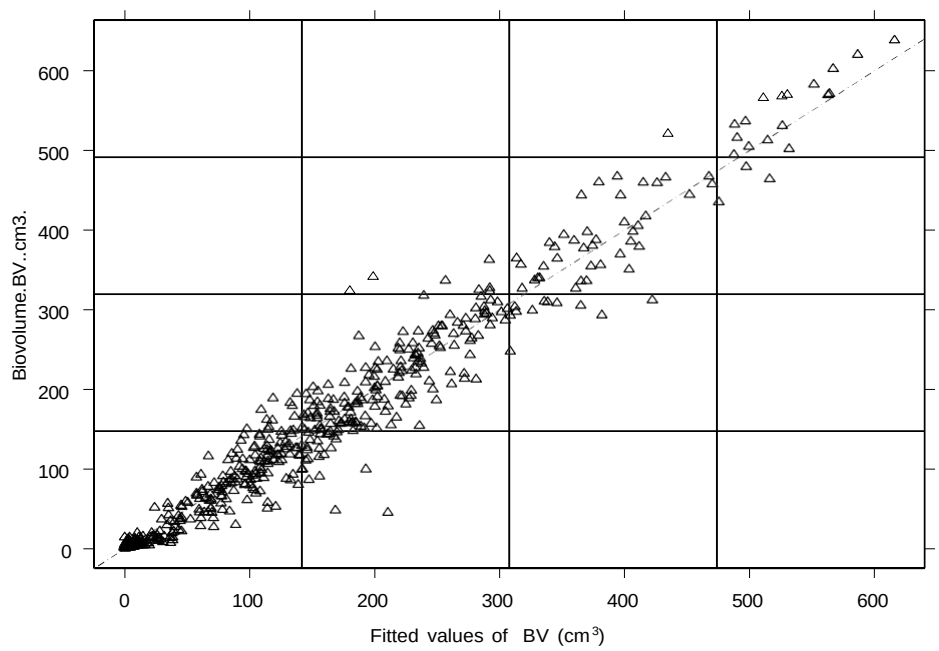

Figure 1. Correlation between observed and predicted maize stalk volume in Sauri, western Kenya

2.535 with a p=0.214 indicated that random effects at treatment plot level on the growth parameters (Gm, Rg and Cg) respectively were not significant at 95%.

Modeling of within-plot variability

Several models were compared; the model with random effects only for the function parameters; (Ty,- maximum growth yield, Rg -rate of growth and Cg- shape of growth curve) as fixed effects (β's) and the model which included exponential of variance-covariance function to correct heteroscedisticity. The analysis of variance of the two models were highly significant with p = 0.0001 favoring the latter model. This agrees with work done by Gregoire *et al.,* (1995) who state that even when using mixed effect model approach the heteroscedisticity in repeated measurements should be modeled. The model was refitted with block-level random effects on Cg parameter omitted after the standard deviation was noted to be negligible (0.00053). The LRT statistic favored the refitted model. Use of AR(1) MA(2) and ARMA(1,1) and other models for correlation structure did not add value to the final model.

Estimates of variance components corresponding to random effects of block-level for the parameters $\hat{\Psi}$ (Gm and Rg), residual error $\hat{\sigma}$ and $\acute{\varepsilon}$ exponential variance structure are shown below.

$$\hat{\Psi} = \begin{pmatrix} 345.231 & 0.0057 \\ 0.0057 & 0.0267 \end{pmatrix}$$

$\hat{\sigma} = 0.7528$

$\acute{\varepsilon} = 0.6578$

Other parameters estimates of fertilizer effects corresponding to fixed effects and their standard errors are summarized in Table 1.

Fertilizer treatment effects on growth functions

Table 1 is a summary of the results of the effects of adding various fertilizer nutrients to the maize growth parameters. Application of N, P and N+P significantly increased maximum growth attained (*Ty,* p = 0.001), while maximum growth attained (biovolume) with P+K was only moderate (p = 0.043). The rate of growth (*Rg*) also showed that while fertilization with N and P increased growth rate significantly (p = 0.001), addition of P, P+K or N+P+K did not have much effect on the rate of maize growth (p = 0.05, Table 1). Significantly, fertilization did not affect the overall shape of the growth curves as shown by the high p-values. This may indicate that the shape of the growth functions is site-independent and does not vary with field conditions and fertilizer treatments. It should also be noted that the fertilizer treatments (K, N+K and N+P+K), had no effect on both maximum growth (biovolume) and rate of growth of the maize stalk volume (Table 1).

A comparison of maize grown on upper lying and low lying topography showed that landform had no effect on variability in maize growth p = 0.752. The sites where N+P fertilizer nutrients were applied had the highest

Table 1. Effects of fertilizer application on maximum growth, growth rate and shape of the maize growth curve in Sauri, western Kenya

Parameters	Fertilizer Treatment	Fixed effect	Estimated Value	Std. Error	F-value	p-value
	Control	β_{01}	154.990	19.088	721.024	0.001
	N	β_{02}	215.183	24.078	103.043	0.001
	P	β_{03}	204.207	24.078	129.592	0.001
	K	β_{04}	144.120	23.078	0.269	0.607
	N+P	β_{05}	257.395	38.108	33.063	0.001
	N+K	β_{06}	164.150	35.109	10.075	0.786
	P+K	β_{07}	177.142	35.109	0.307	0.046
(Ty) $Growth_{max}$ Yield	N+P+K	β_{08}	148.046	55.152	1.324	0.249
	Control	β_{11}	0.536	0.047	540.386	0.001
	N	β_{12}	0.577	0.024	.170	0.001
	P	β_{13}	0.584	0.017	0.334	0.001
	K	β_{14}	0.533	0.018	2.896	0.088
	N+P	β_{15}	0.598	0.031	0.698	0.403
	N+K	β_{16}	0.543	0.041	1.418	0.254
	P+K	β_{17}	0.576	0.025	3.805	0.084
(Rg) Growth rate	N+P+K	β_{18}	0.544	0.042	8.473	0.093
	Control	β_{21}	6.935	1.122	335.131	0.001
	N	β_{22}	7.634	0.466	0.2016	0.899
	P	β_{23}	6.058	0.195	0.165	0.068
	K	β_{24}	6.072	1.207	0.724	0.394
	N+P	β_{25}	6.742	0.528	0.406	0.524
	N+K	β_{26}	6.571	0.753	0.906	0.795
	P+K	β_{27}	6.102	0.275	0.604	0.083
(Cg) Shape of Growth curve	N+P+K	β_{28}	6.747	0.649	0.464	0.049

Where: β_s = Estimated parameters of the effect of corresponding fertilizer nutrients on maize growth; Ty = Estimated maximum attainable growth yield as influenced by fertilizer treatments applied; Rg = Estimate rate of growth of maize as affected by corresponding fertilizer treatments; and Cg = Shape of the curve constant associated with plant species allometric constant.

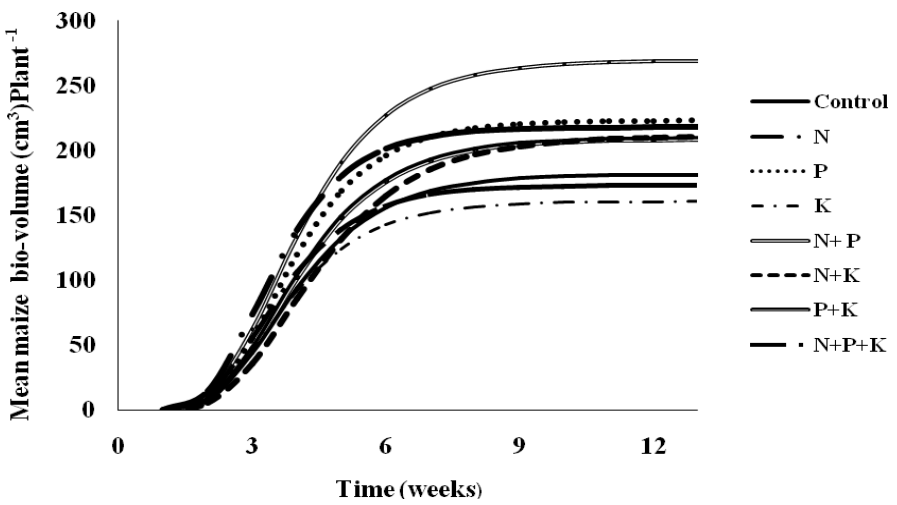

Figure 2 Effects of fertilizer application on maize stalk volume (biovolume) during the growing season in Sauri, western Kenya

maximum attainable growth (Figure 2). N+K and P+K fertilizer treatments had similar maximum attainable growth yields which were lower than those of N-only and P-only fertilizer treatments (Figure 2) probably because of

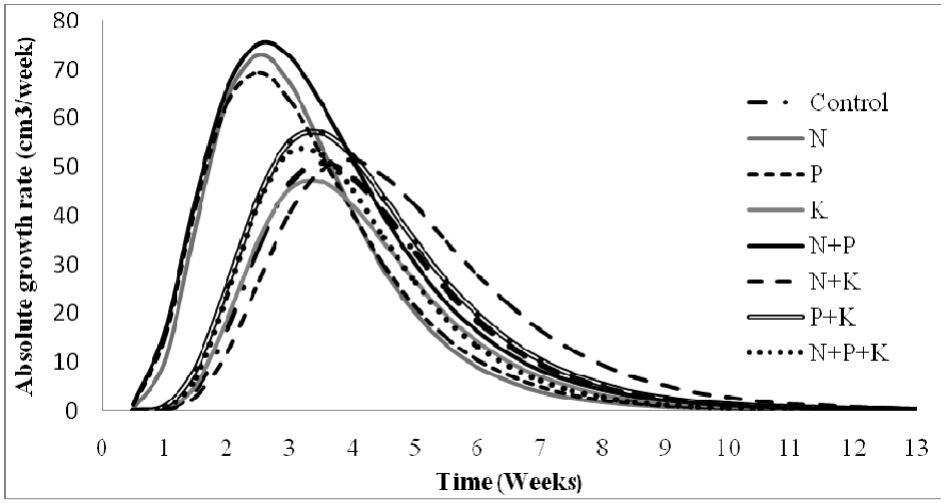

Figure 3. Observed variation of absolute growth rate of maize biovolume due to effects of applied fertilizer treatments in Sauri, western Kenya

Table 2. Effect of fertilizer treatments on the time maximum absolute growth rate is attained during maize growth in Sauri, western Kenya

Weeks	Control	N	P	K	N+P	N+K	P+K	N+P+K
1	0.55	10.06	15.10	0.77	16.07	0.52	1.05	0.62
2	17.06	63.75	63.74	18.63	66.79	12.39	25.49	23.58
3	46.60	67.31	69.19	45.53	73.35	40.71	55.96	42.06
4	47.03	41.19	40.11	46.15	51.21	51.92	56.05	45.07

antagonistic interactions between K and both N and P. On the other hand there was a synergistic interaction between N and P as exemplified by the higher growth of N+P than the individual N or P treated crops (Figure 2). Interestingly, N+P+K addition reduced growth to much lower than N+P application, confirming earlier observations regarding possible antagonistic effects of K on N and P. Maize growth did not respond to K fertilizer treatment since the yields were even lower than the control, though not significant.

Figure 3 shows that growth rate varied both in time and magnitude depending on the fertilizer treatment. Thus, while the control had the least maximum growth rate, the N+P treatment that had the highest. It is also evident that the control, K, N+K, N+P+K and P+K achieved maximum growth rate one week after, compared to N, P, and N+P treatments (Table 2).

Comparison of maize growth response to fertilizer addition across the nine villages in relation to the initial status (control) of the site quality (Figure 4) showed that Nyamboga village as the poorest site compared to Sauri village consistently showed the lowest maize growth compared to the other villages, with or without fertilizer application. Response to fertilizer application suggested that single N or P fertilization did not eliminate nutrient limitation, while application of N+P gave the highest response in all individual fields studied suggesting positive interactions between these nutrients. The high variations in growth responses depicted across the villages in Figure 4 suggests that blanket fertilizer recommendations in the area may be inappropriate for intensive farm management, thus the need to treat each farm field as a distinct management unit. Correlation analysis of the maximum stalk volume growth (biovolume) of the maize indicated that it predicted grain yield sufficiently (Figure 5) giving a high correlation coefficient ($r^2 = 0.70$; $p = 0.035$). The rate of growth of the maize stalk and shape of the growth curves parameters showed no correlation with maize grain yield.

DISCUSSION AND CONCLUSION

Results from this study have demonstrated that maximum growth attained by the maize crop has a strong correlation with grain yield, thus it may be a good indicator for predicting crop response to fertilizer addition. The difference in the magnitude of the responses suggests that soil fertility status influenced maize growth patterns differently, as was also observed by Martin (1987) and Garcia (2004). The correlation between maize stalk volume and yield indicated that N+P fertilizer

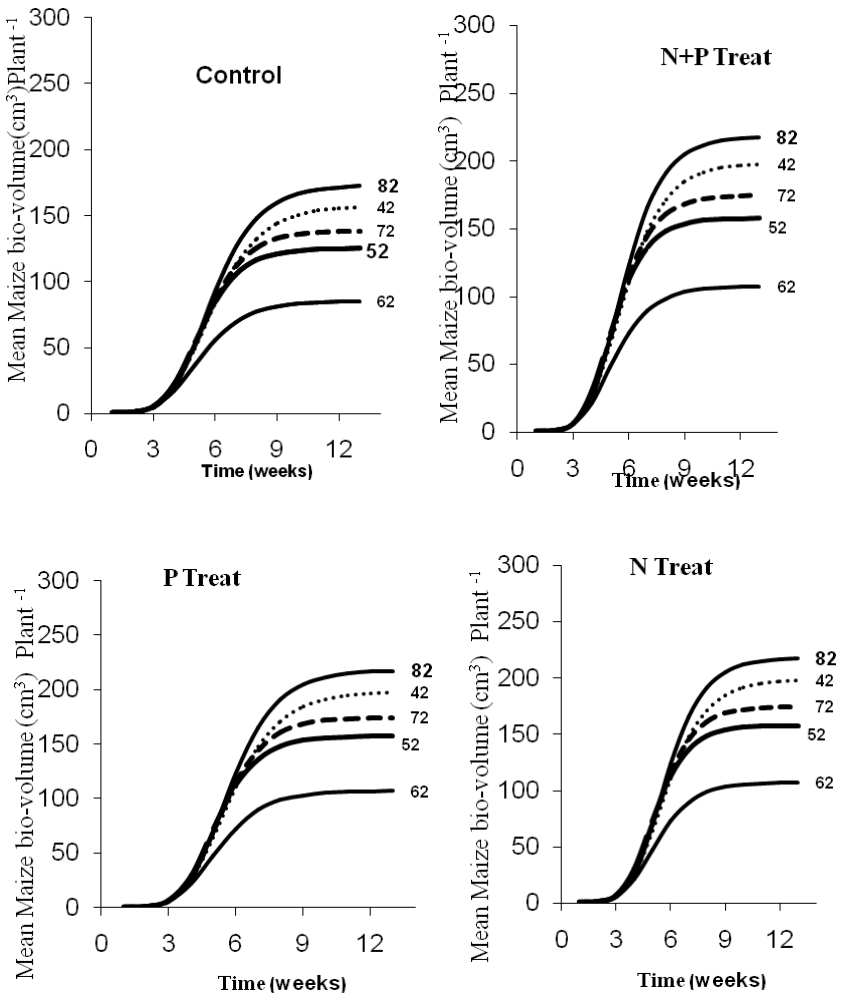

Figure 4: Effects of various fertilizer treatments in relation initial soil fertility status (Control) on maize in Sauri, western Kenya
Key: Flat plots
82-Sauri-B, 42-Nyamninia – B, 72-Kosoro, 52-Silula, 62-Nyamboga

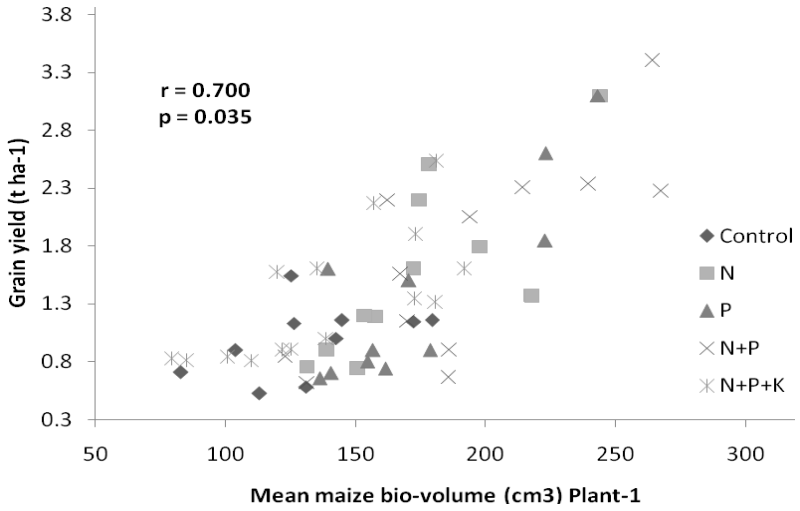

Figure 5. Correlation between maize grain yield and maximum stalk-volume growth in various villages in Sauri, western Kenya

combination response gave the greatest grain yield as compared to control. These results compare well with findings by and Gikonyo and Smithson (2001) who reported lack of responses to K in final maize dry matter on slightly acidic granitic and phonolithic soils that were similar to the soils of Sauri (Gikonyo *et al*., 2000). It is evident that the effects of N and P on plant growth are partly additive, and that growth limitation by one of these nutrients reduces the efficiency of use of the other. Nitrogen-only addition resulted in the highest stalk volume and growth rate compared to P only. Bottcher and Rhue (2000) also observed that the shape and slope of individual N and P growth-response curves usually differ only in magnitude and scale, with the former showing a stronger potential growth response than the latter, even though maize response to P is generally not as consistent and dramatic as response to N fertilization.

The study also demonstrated that soil fertility status influences maximum attainable yield and growth rate, but not shape of the growth curves. The finding that biovolume had significant correlation with maize grain yield may indicate that nutrients; availability, synergy and antagonism influence both maize stalk growth and grain yield.

ACKNOWLEDGEMENTS

We would like to acknowledge the generous assistance of Mr. Paul Abuom of ICRAF during data collection and Dr. C. T. Omuto for helping in the formulation of the statistical model in S-plus®.

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